

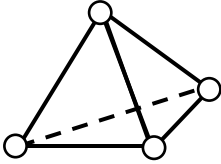
A Low-cost and Pressure-checker-free Smoothed Finite Element Formulation using 4-node Tetrahedral Meshes for Nearly Incompressible Large Deformation Analyses

Yuki ONISHI
(Institute of Science Tokyo, Japan)

Motivation

What we want to do:

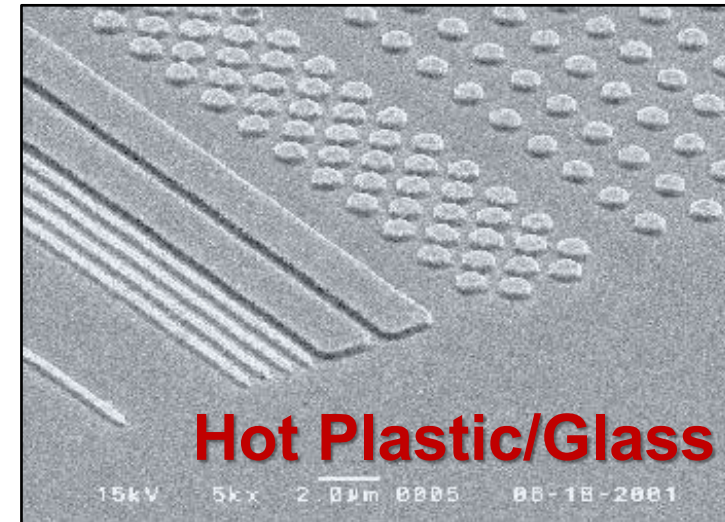
- Solve **severe large deformation** analyses with finite elements.
- Treat complex geometries with **tetrahedral (TET) meshes**.
- Consider **nearly incompressible materials** ($\nu \simeq 0.5$).
- Support **contact** problems.
- Handle **auto re-meshing**.



Current situation:

There is no **triple-threat** TET element that satisfies

1. **Good accuracy,**
2. **Fast compute time, and**
3. **Enough robustness in large deformation.**



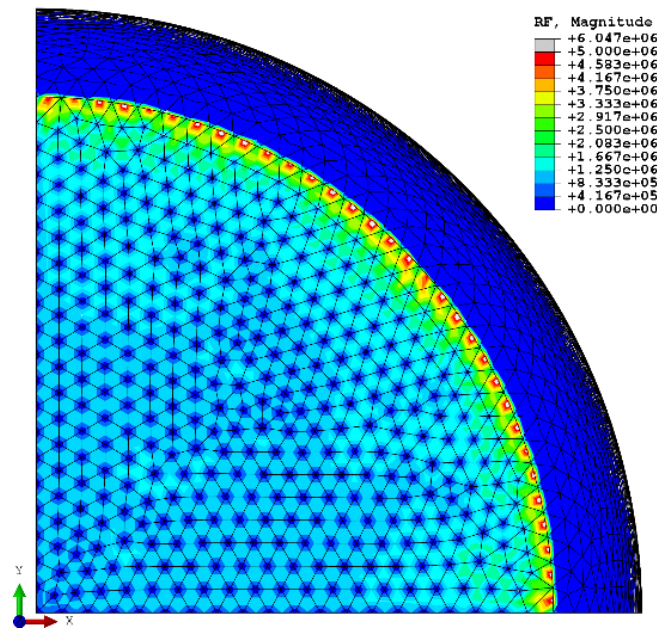
Issues of TET Elems in Accuracy and Their Solutions

Issues in Accuracy	Solutions
Shear Locking	2 nd -order, ES-FEM, NS-FEM, etc.
Volumetric Locking	$u - p$, SRI, B-bar, F-bar, NS-FEM, etc.
Reaction Force Oscillation	1 st -order, Modified 2 nd -order, etc.
Pressure Checkerboarding	None! (No absolute solution)

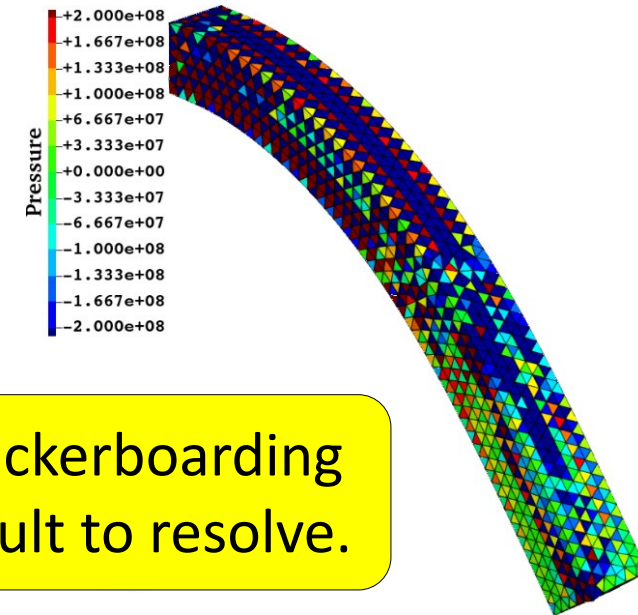
S-FEMs will be detailed later.

Related to Nodal Force/Disp.

Related to Stress/Strain



Typical reaction force oscillation (ABAQUS C3D10)



Pressure checkerboarding is quite difficult to resolve.

Typical pressure checkerboarding (ABAQUS C3D4)

Develop a new **smoothed finite element method** with **4-node tetrahedral** meshes (**ES-FEM-SRI-T4**) to **suppress pressure checkerboarding** in nearly incompressible large deformation analysis.

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- Method: Brief of ES-FEM-SRI-T4
- Result & Discussion 1: Performance of ES-FEM-SRI-T4
- Result & Discussion 2: Performance of Conventional S-FEM with Post-Smoothing in Pressure
- Summary

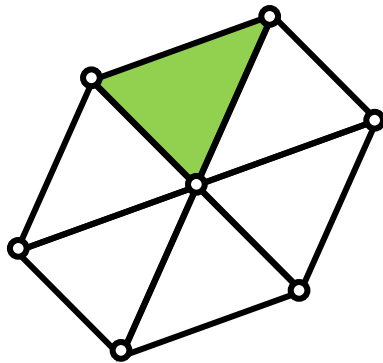
Quick Introduction to S-FEM

What is S-FEM?

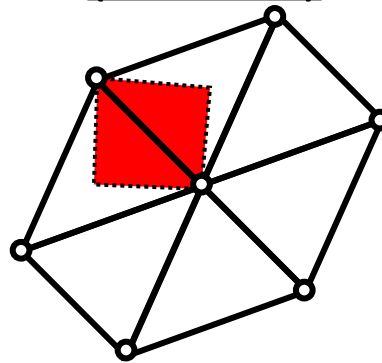
- **Smoothed** finite element method is an FE formulation proposed by Prof. G.R. Liu in 2006.
- After 20 years of active research, practical applications are now underway in various fields.
- S-FEM is one of the **gradient (strain) smoothing** techniques.
- There are **many kinds of S-FEMs** depending on the scheme of smoothing.
- There are a few classical S-FEMs depending on the smoothing domain.

For example, in a 2D triangular mesh:

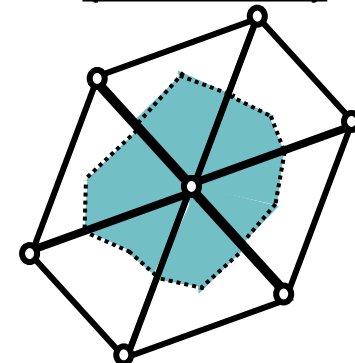
Standard FEM



Edge-based S-FEM
(ES-FEM)



Node-based S-FEM
(NS-FEM)



Each colored area shows the domain for gradient smoothing.

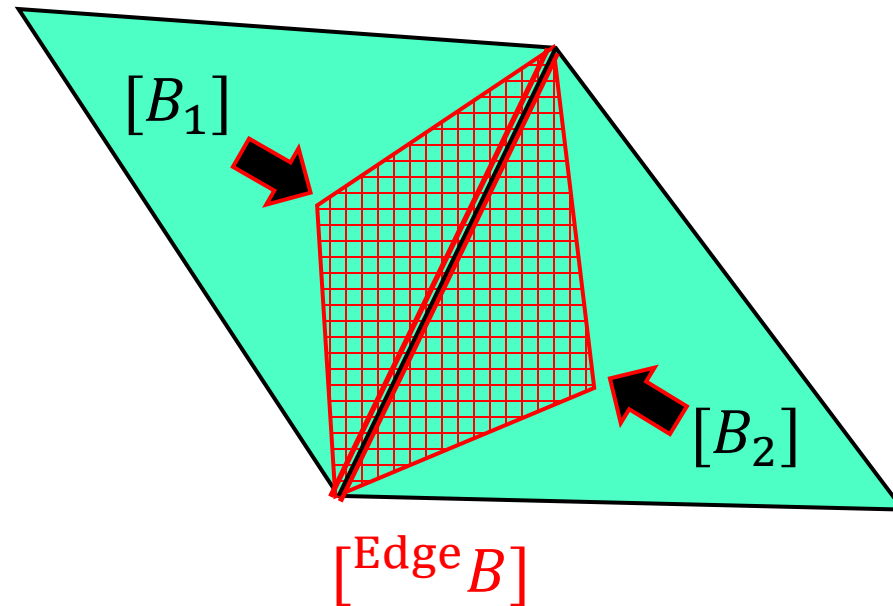
Brief of ES-FEM

Let us consider a mesh with only two 3-node triangular (T3) cells.

- Calculate $[B](=dN/dx)$ at each cell as usual.
- Distribute each $[B]$ to the connecting **edge** with an area weight and build $[^{\text{Edge}}B]$.
- Calculate strain (ϵ), Cauchy stress (σ) and nodal internal force $\{f^{\text{int}}\}$ in each **edge smoothing domain** with $[^{\text{Edge}}B]$.

Let me
explain in 2D
for simplicity

As if putting
a Gauss point
on each edge center



$[^{\text{Edge}}B]$

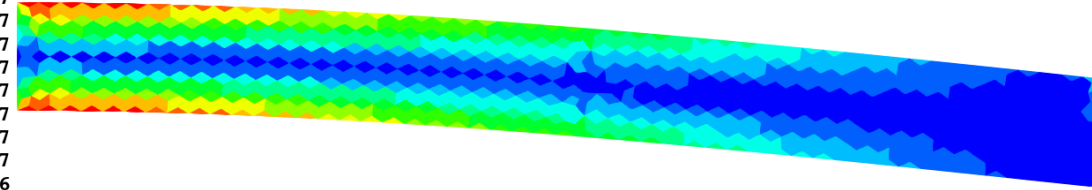
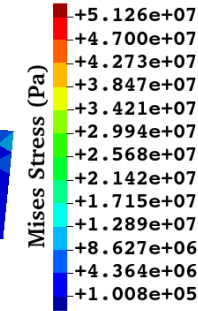
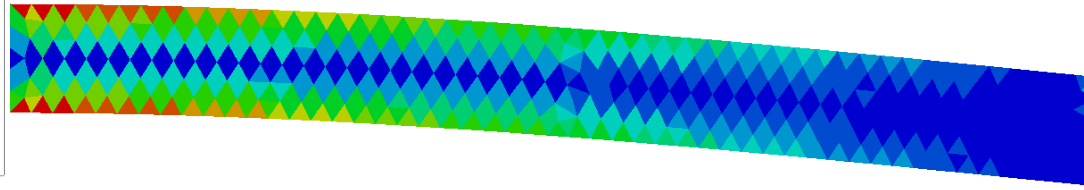
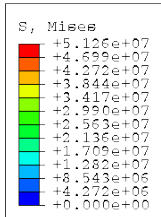


Edge ϵ , Edge σ , $\{\text{Edge } f^{\text{int}}\}$ etc.

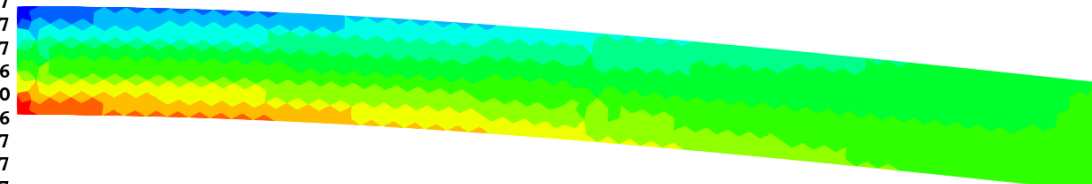
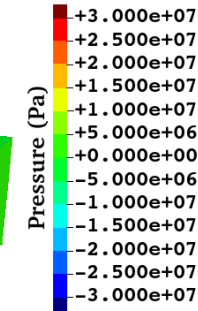
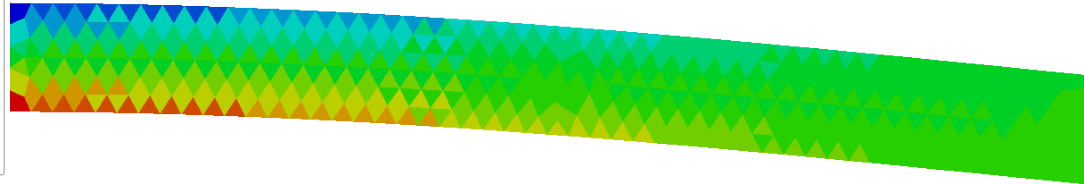
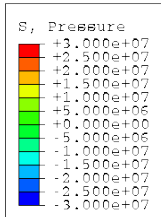
Performance of ES-FEM

Cantilever Bending Analysis with Dead Load at the Tip Plane-Strain, Size:10x1 m, $\nu = 0.3$

Mises stress



Pressure



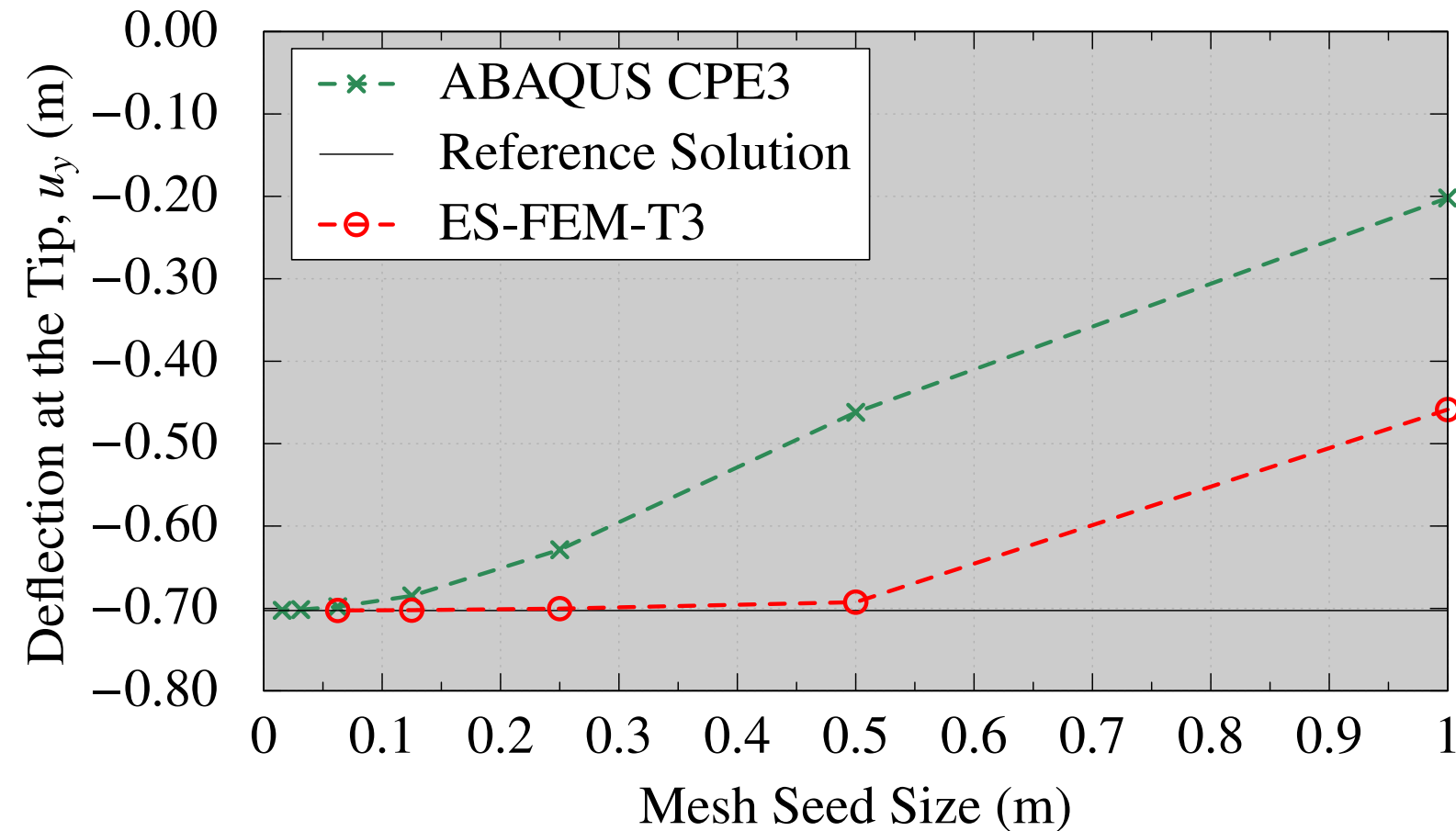
ABAQUS CPE3
(Standard FEM-T3)

ES-FEM-T3

ES-FEM-T3 gives more accurate stress/strain distributions than the standard FEM-T3 using the same mesh.

Performance of ES-FEM

Cantilever Bending Analysis with Dead Load at the Tip Plane-Strain, Size:10x1 m, $\nu = 0.3$



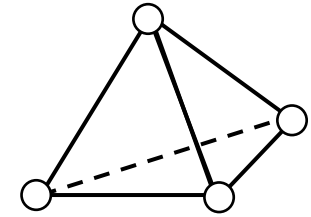
Mesh Seed Size (m)	The Number of Cells
1	22
1/2	84 Converged
1/4	342
1/8	1,442
1/16	5,766 Converged
1/32	23,034
1/64	92,146

ES-FEM-T3 is shear locking free and thus the mesh convergence rate of displacement/force is much faster than the Standard FEM-T3.

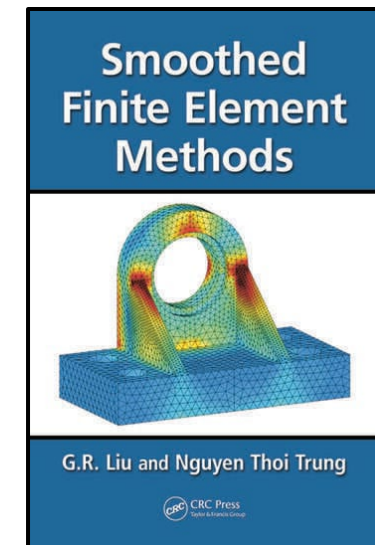
What are the benefits of S-FEM?

1. **Super-linear mesh convergence rate with T4 mesh.**
(Almost same rate as 2nd-order T10 elements.)
2. **Shear locking free with ES-FEM-T4.**
(Good accuracy with T4 mesh in solid mechanics.)
3. **Volumetric locking free with NS-FEM-T4.**
(Key technique for rubber-like nearly incompressible solid.)
4. **Little accuracy loss with skewed meshes.**
(No problem with complex geometry or severe deformation.)
5. **No increase in DOF.**
(Purely displacement-based formulation.)
6. **Easy to code.**
(keeping away from mixed variational formulations.)

T4: 4-node Tetrahedra



T4



G.R. Liu
et al.,
CRC Press

Search

S-FEM wiki

S-FEM is a powerful method suitable for practical industrial applications.

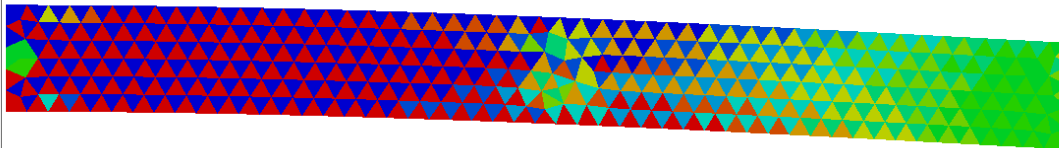
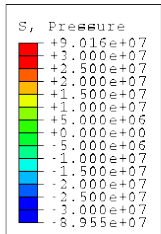
Two Major Issues in ES-FEM

In **nearly incompressible** solid, ES-FEM-T3 causes

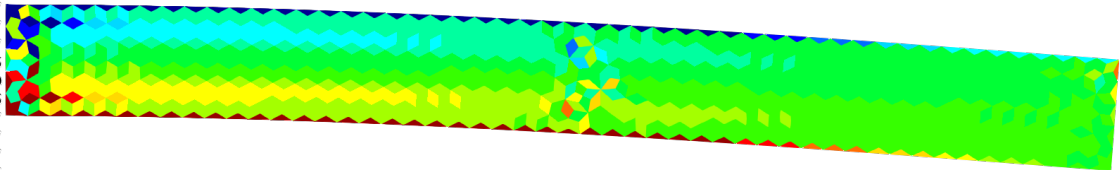
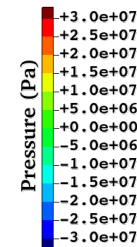
- Volumetric locking
- Pressure checkerboarding

Same Cantilever Bending Analysis with $\nu = 0.49$

Pressure



ABAQUS CPE3
(Standard FEM-T3)



ES-FEM-T3

A brand-new code with a novel S-FEM formulation
for **nearly incompressible** solids is still awaited.

Method

Brief of ES-FEM-SRI-T4

Brief of ES-FEM

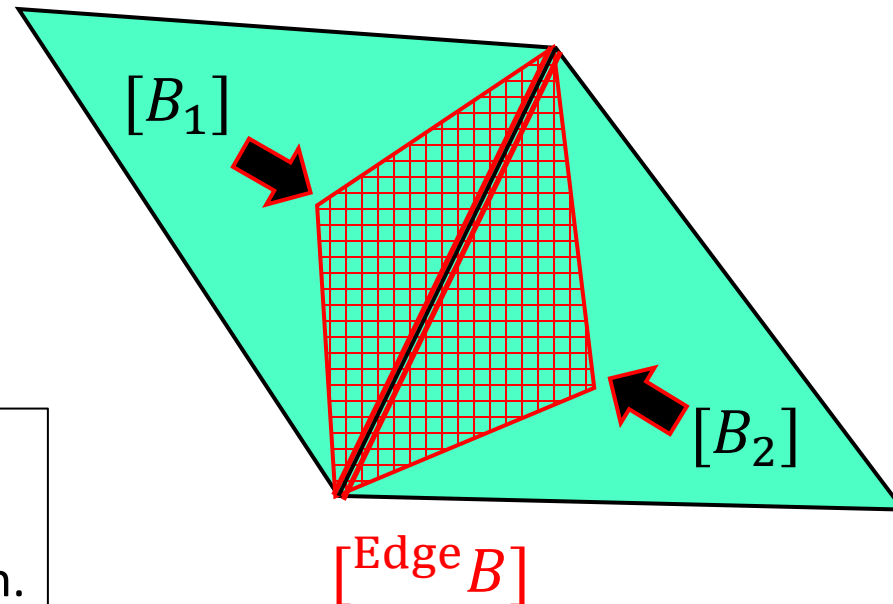
Let us consider a mesh with only two 3-node triangular (T3) cells.

Let me
explain in 2D
for simplicity

- Make $[B](=dN/dx)$ at each cell as usual.
- At each **edge**, gather $[B]$ s of the connecting cells and average them with area weights to build $[^{\text{Edge}}B]$.
- Calculate strain (ϵ), stress (σ) and nodal internal force $\{f^{\text{int}}\}$ in each **edge smoothing domain** with $[^{\text{Edge}}B]$.

As if putting
a Gauss point
on each edge center

Strain distribution is
piecewise constant
in each smoothing domain.



No shear locking.

Cannot avoid
volumetric locking &
pressure checkerboarding.

$\downarrow$ Edge ϵ , Edge σ , $\{\text{Edge } f^{\text{int}}\}$ etc.

Brief of NS-FEM

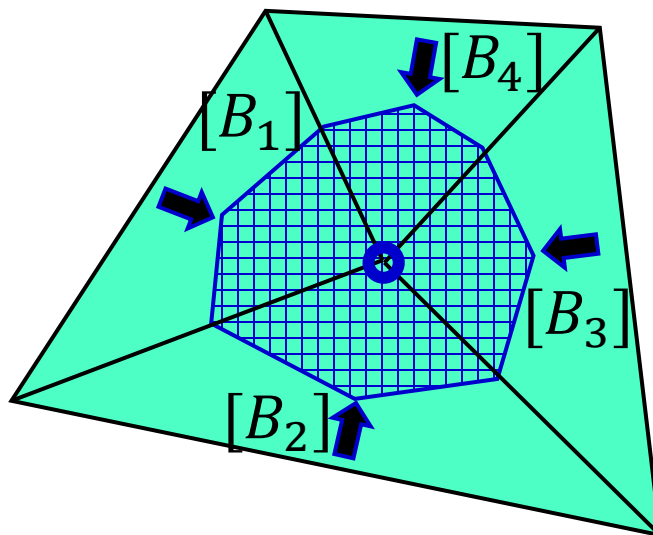
Let us consider a mesh with only four 3-node triangular (T3) cells.

- Make $[B](=dN/dx)$ at each cell as usual.
- At each **node**, gather $[B]$ s of the connecting cells and average them with area weights to build $[^{\text{Node}}B]$.
- Calculate strain (ϵ), stress (σ) and nodal internal force $\{f^{\text{int}}\}$ in each **nodal smoothing domain** with $[^{\text{Node}}B]$.

Let me
explain in 2D
for simplicity

As if putting
a Gauss point
on each node center

Strain distribution is
piecewise constant
in each smoothing domain.



No shear/volumetric locking.
Minor pressure checkerboarding.

Cannot avoid
spurious low-energy modes.

$[^{\text{Node}}B]$
Node ϵ , Node σ , $\{^{\text{Node}}f^{\text{int}}\}$ etc.

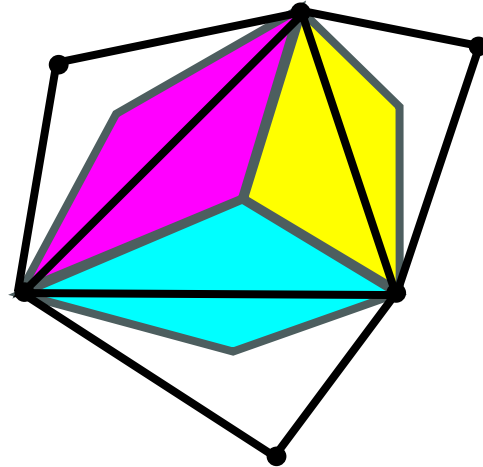
Brief of **SelectiveES/NS-FEM** (Conventional Method)

Apply the **selective reduced integration (SRI)** to ES-FEM to handle **rubber-like solids**

Let me
explain in 2D
for simplicity

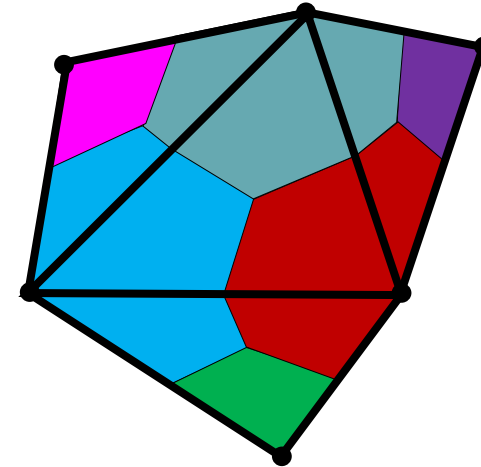
Deviatoric
Part

(1) Calculate ε^{dev} at each edge
with ES-FEM



(2) Calculate σ^{dev} at each edge
and its contribution to $\{f^{\text{int}}\}$

(3) Calculate ε^{vol} at each node
with NS-FEM



(4) Calculate σ^{hyd} at each node
and its contribution to $\{f^{\text{int}}\}$

Volumetric
Part

**Selective Reduced
Integration (SRI)**

- No shear locking.
- Good accuracy in deviatoric stress.

- No volumetric locking.
- Unacceptable pressure checkerboarding.

(5) Assemble $\{f^{\text{int}}\}$

Brief of ES-FEM-SRI (New Method)

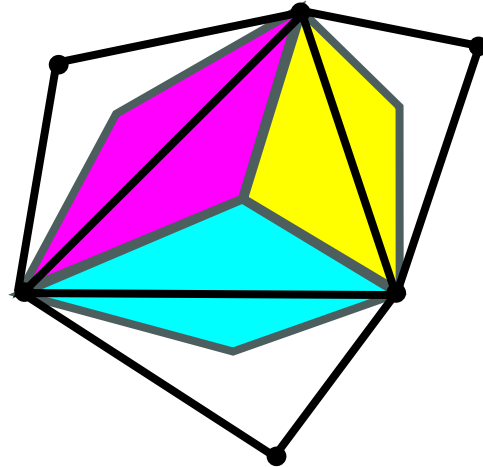
Apply the **selective reduced integration (SRI)** to ES-FEM to handle **rubber-like solids**

Let me
explain in 2D
for simplicity

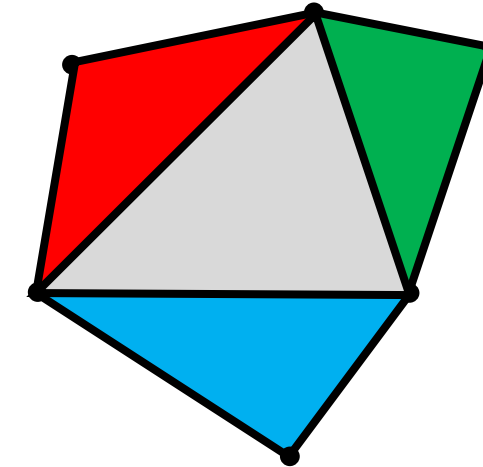
(1) Calculate ε^{dev} at each edge
with ES-FEM

(3) Calculate ε^{vol} at each **cell**
with **double strain smoothing**

Deviatoric
Part



(2) Calculate σ^{dev} at each edge
and its contribution to $\{f^{\text{int}}\}$



Volumetric
Part

Double strain smoothing:
Strain smoothing at
nodes (NS-FEM) followed
by another smoothing at
cells (NS-FEM⁻¹)

(4) Calculate σ^{hyd} at each **cell**
and its contribution to $\{f^{\text{int}}\}$

**Selective Reduced
Integration (SRI)**

- No shear locking.
- Good accuracy in deviatoric stress.

- No volumetric locking.
- **Less pressure checkerboarding.**

(5) Assemble $\{f^{\text{int}}\}$

CPU Time of S-FEM-T4s

- Since the most of CPU time for implicit analyses is spent solving the stiffness equation (i.e., $[K]\{u\} = \{f\}$), the size of $[K]$ matrix (N) directly affects the CPU time.
- S-FEMs are purely displacement-based FE formulations; thus, **the matrix size (N) is exactly identical to that of FEM-T4.**
- Both the **SelectiveES/NS-FEM-T4** and **ES-FEM-SRI-T4** have the same **bandwidth of $[K]$** , which is **x4.3 wider than that of FEM-T4**, meaning about x4 slower than FEM-T4 w/ the same mesh.

Formulation	Bandwidth of $[K]$	v.s. FEM-T4 Ratio
FEM-T4	14 nodes x 3 DOF	1
FEM-T10	28 nodes x 3 DOF	2.0
ES-FEM-T4	45 nodes x 3 DOF	3.2
NS-FEM-T4, SelectiveES/NS-FEM-T4, ES-FEM-SRI-T4	60 nodes x 3 DOF	4.3

Remind that S-FEM-T4s have much faster mesh convergence rate than FEM-T4.
Therefore, S-FEM-T4s are faster than FEM-T4 to obtain a solution with a certain level of accuracy.

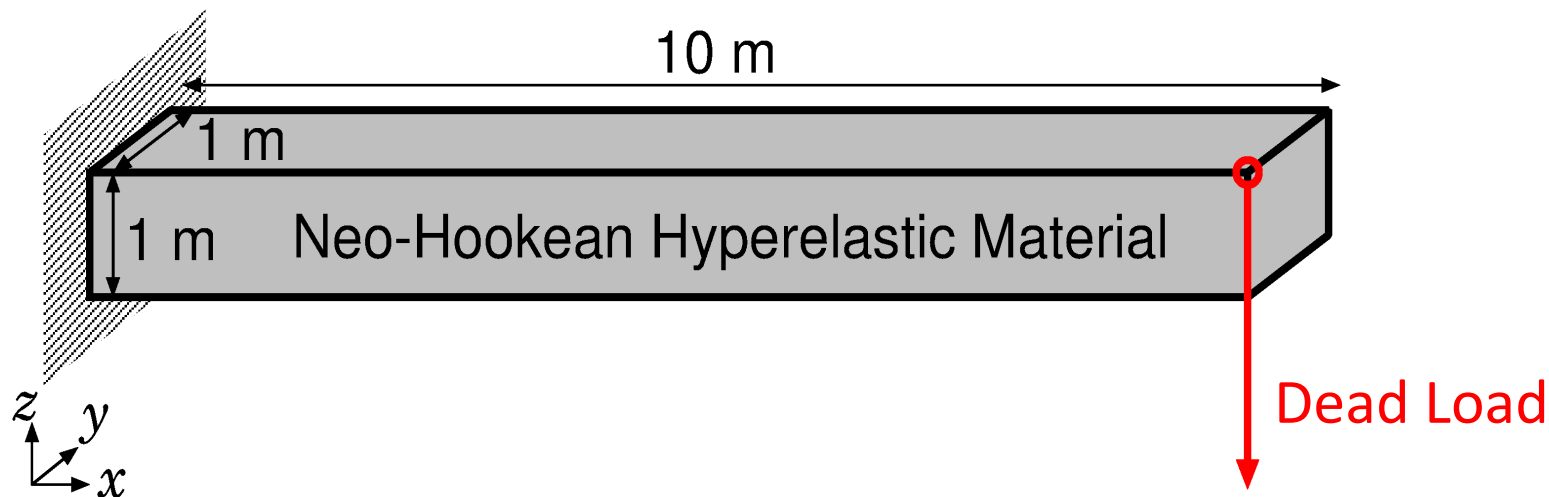
ES-FEM-SRI-T4 costs the same as **SelectiveES/NS-FEM-T4** and is relatively low-cost.

Result & Discussion 1

Performance of ES-FEM-SRI-T4

Bending of Rubber Cantilever

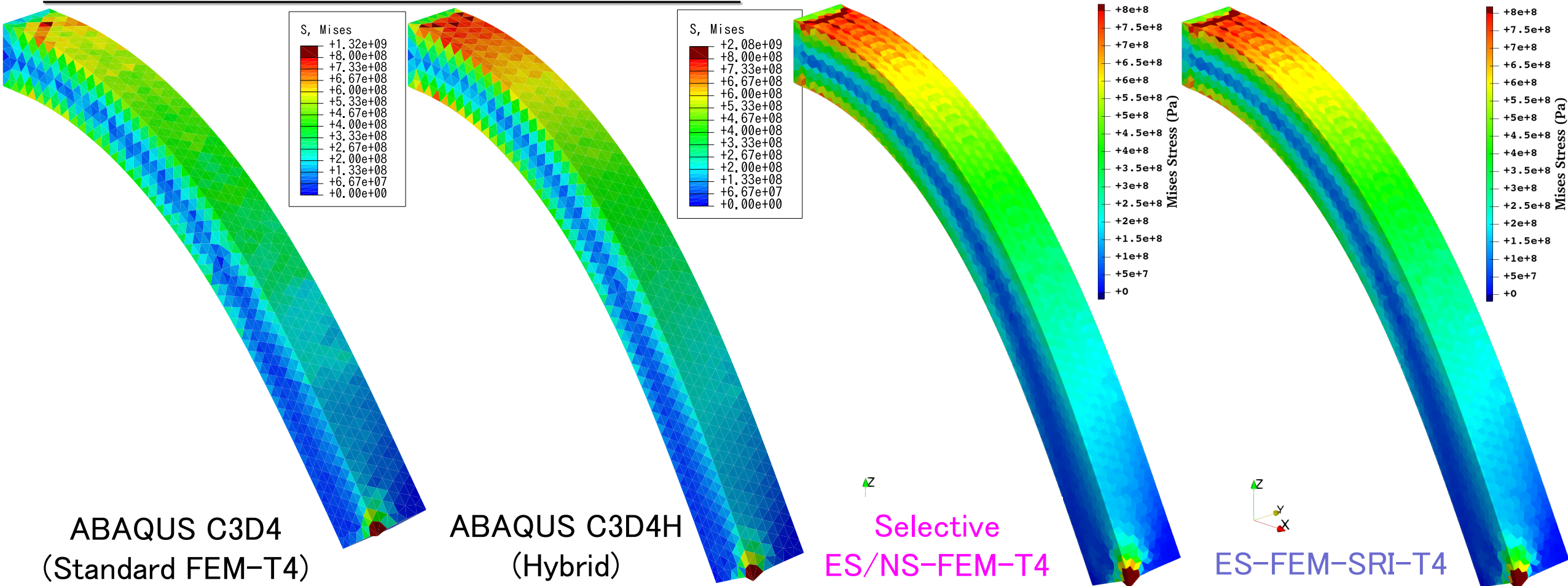
Outline



- 10 x 1 x 1 m cantilever.
- Neo-Hookean hyperelastic material, $E_{ini} = 6$ GPa, $\nu_{ini} = 0.49$.
- Dead load applied to the tip node.
- A large deflection analysis with $u_z = -6.5$ m at the final state.
- Compared the results of **ABAQUS C3D4**, **ABAQUS C3D4H**, **SelectiveES/NS-FEM-T4**, and **ES-FEM-SRI-T4**.

Bending of Rubber Cantilever

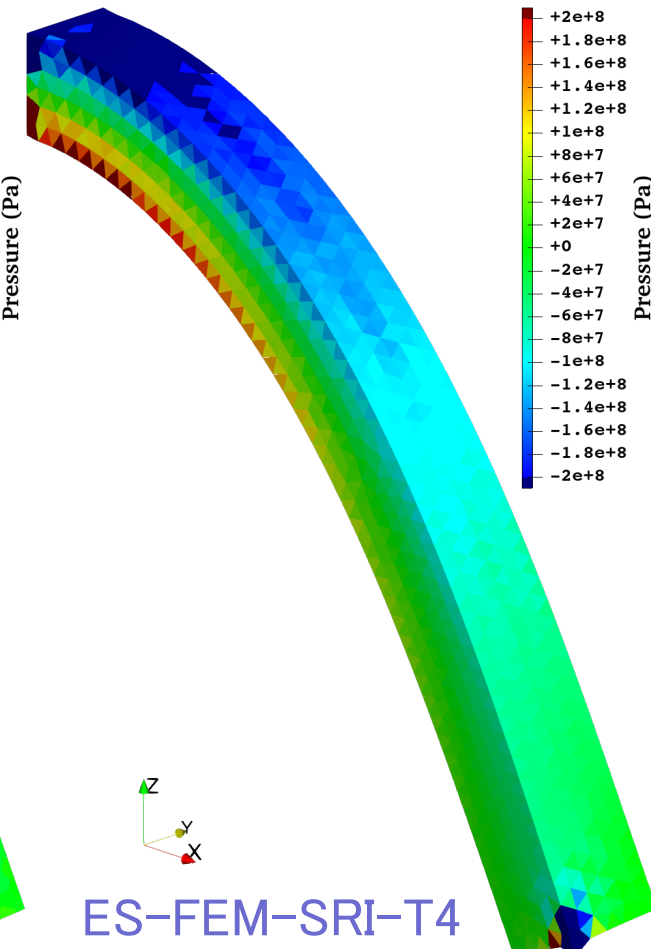
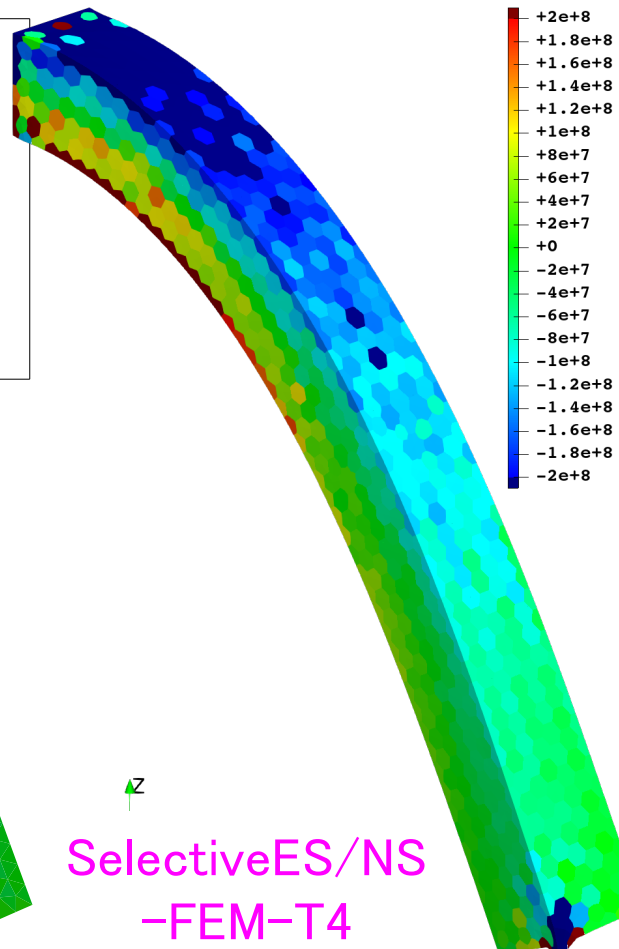
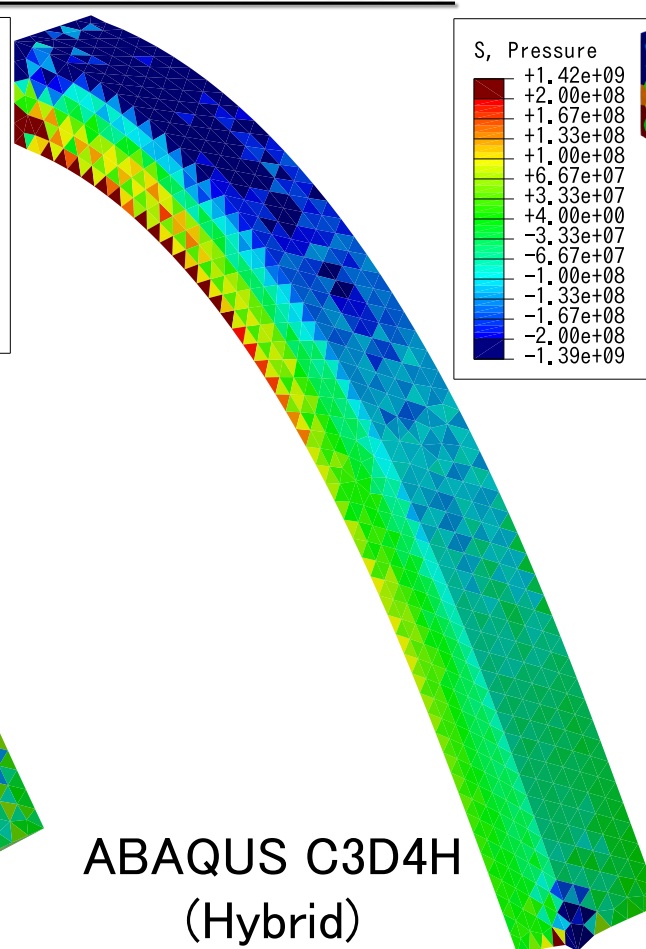
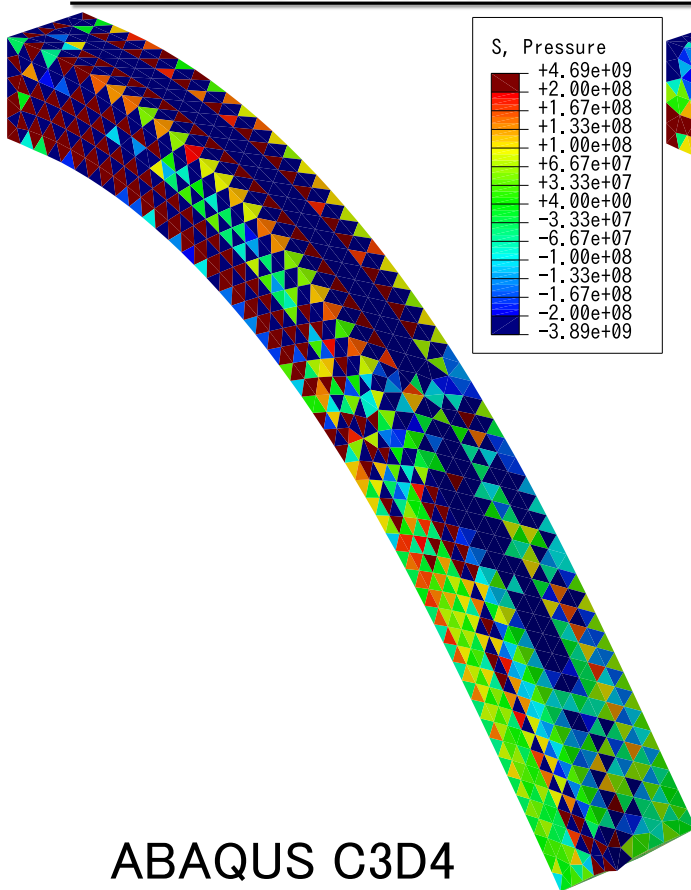
Mises Stress Distribution at the Final State



- C3D4 suffers from severe volumetric locking, whereas C3D4H has slight shear locking.
- **Selective ES/NS-FEM-T4** and **ES-FEM-SRI-T4** are locking-free and present smoother stress distributions.

Bending of Rubber Cantilever

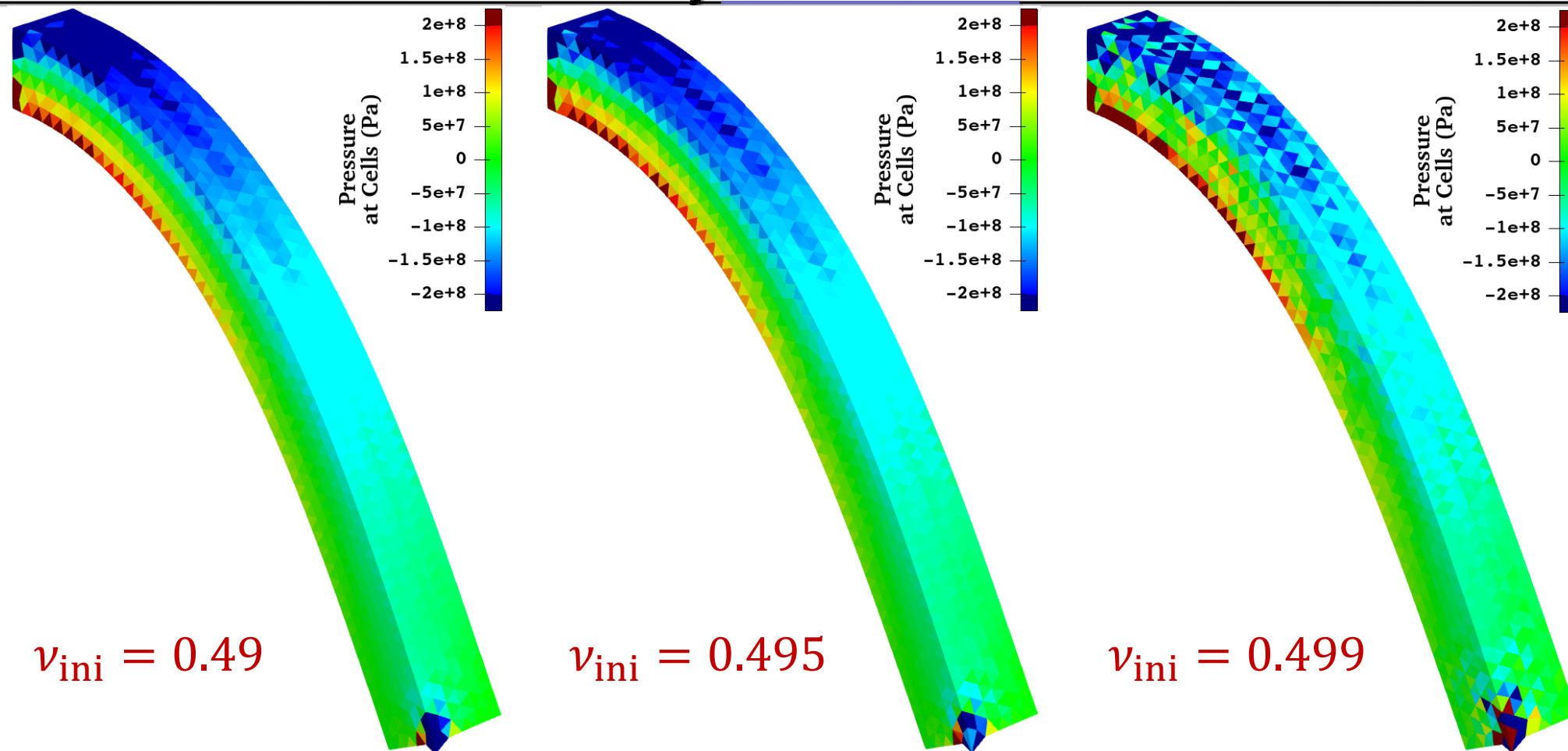
Pressure Distribution at the Final State



- Only ES-FEM-SRI-T4 shows an acceptable pressure-checker-free result.
- Inaccurate pressure leads to accuracy loss in σ_{ii} , direction and magnitude of principal stresses, etc..

Bending of Rubber Cantilever

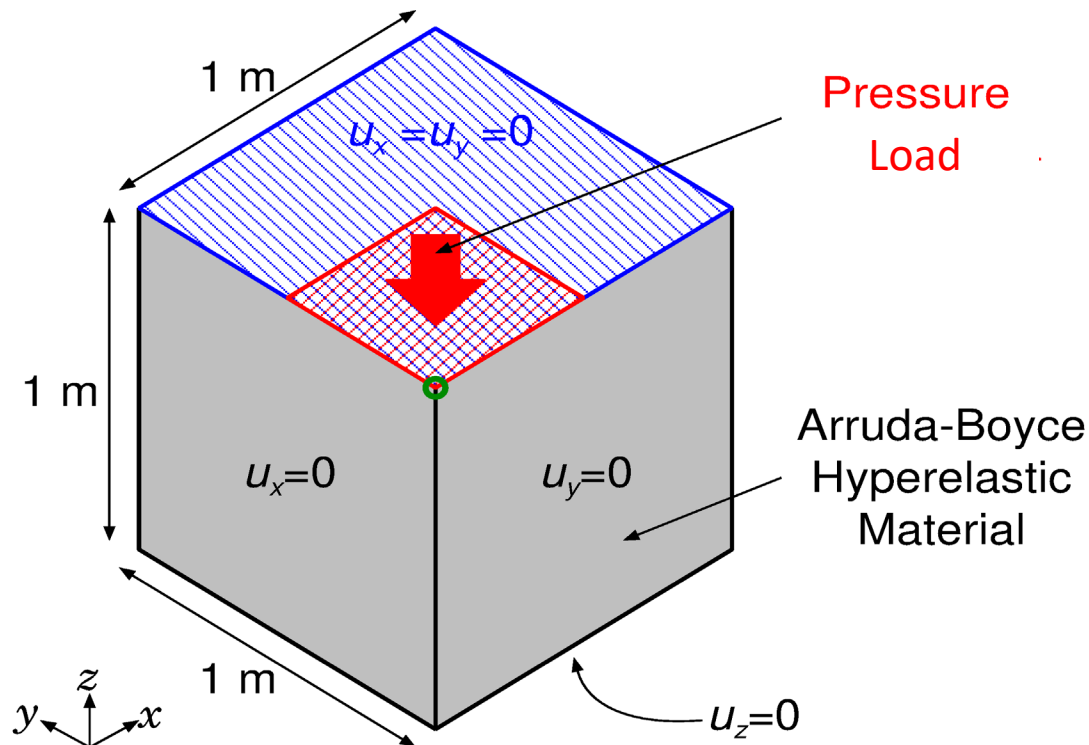
*Pressure Distribution at the Final State of **ES-FEM-SRI-T4** with Various Poisson's Ratios*



ES-FEM-SRI-T4 is pressure-checker-free up to $\nu = 0.49$, but not beyond that.

Pressuring of Rubber Block

Outline

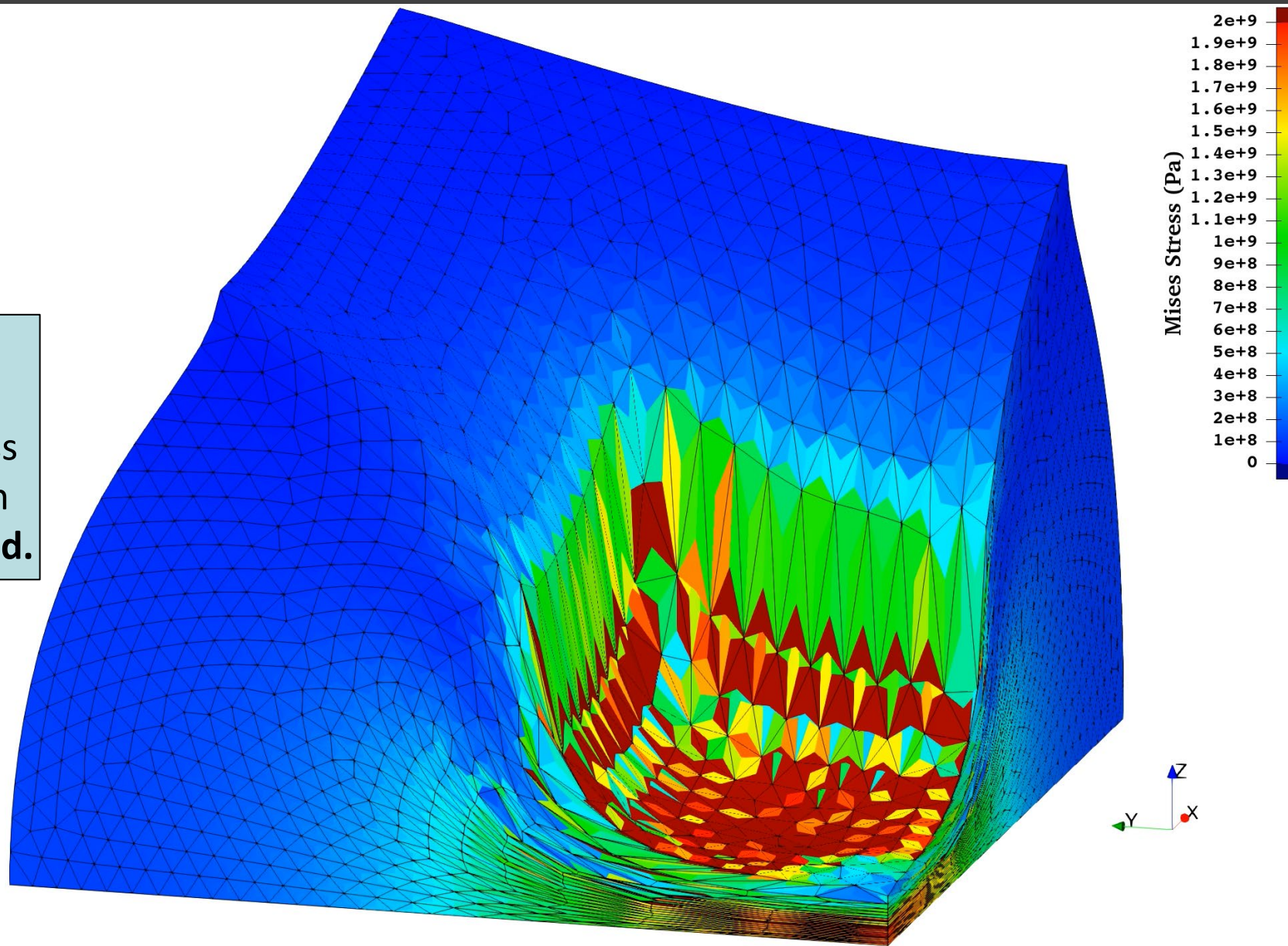


- 1 x 1 x 1 m block.
- Arruda-Boyce hyperelastic material, $E_{ini} = 24$ GPa, $\nu_{ini} = 0.49$.
- Applying pressure on $\frac{1}{4}$ of the top face with lateral confinement.
- Evaluated the result of [ES-FEM-SRI-T4](#).

Pressuring of Rubber Block

Mises Stress Animation of ES-FEM-SRI-T4

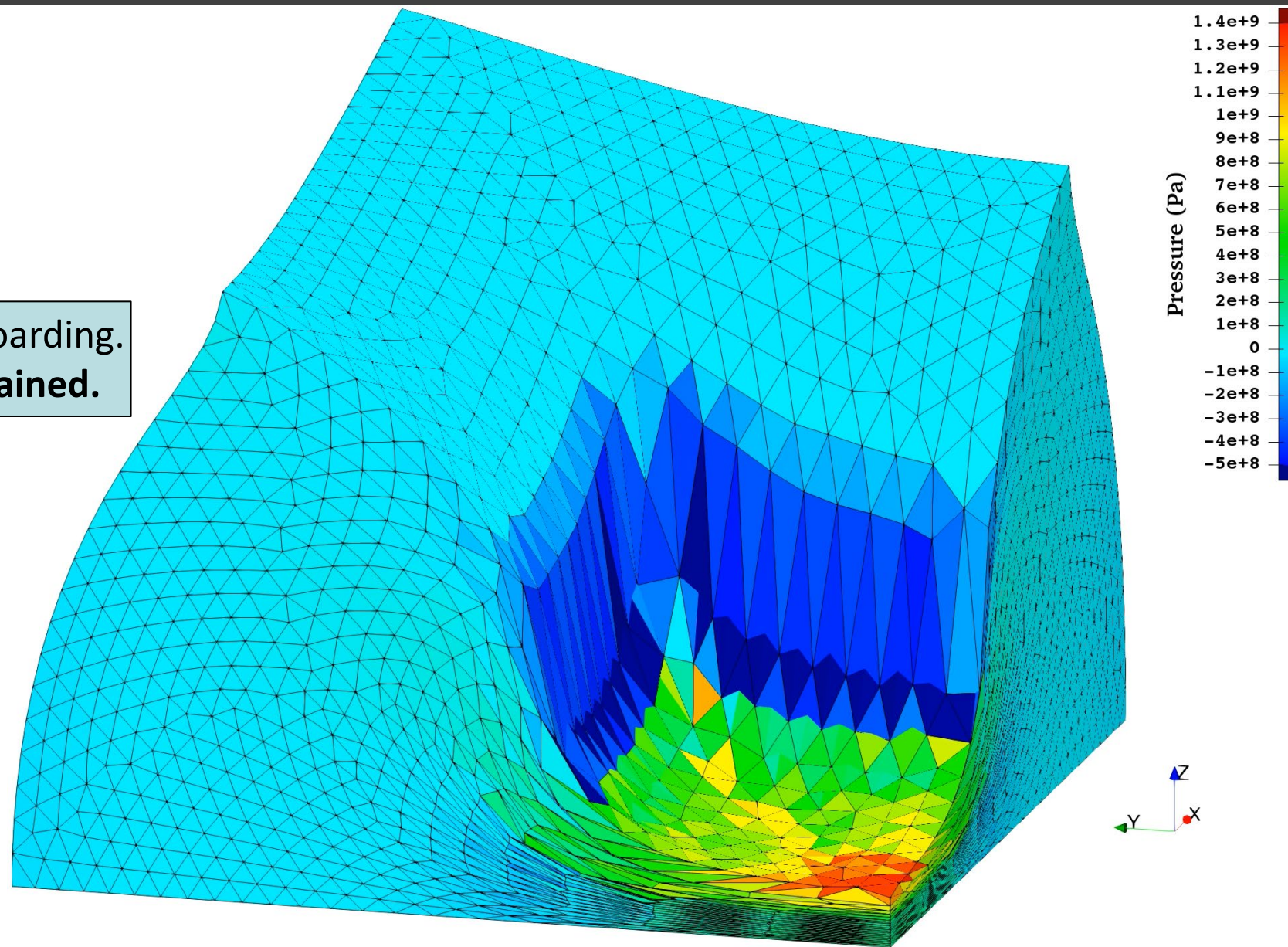
- Smooth distribution of Mises stress.
 - Sufficient robustness in large deformation
- No major issue is found.**



Pressuring of Rubber Block

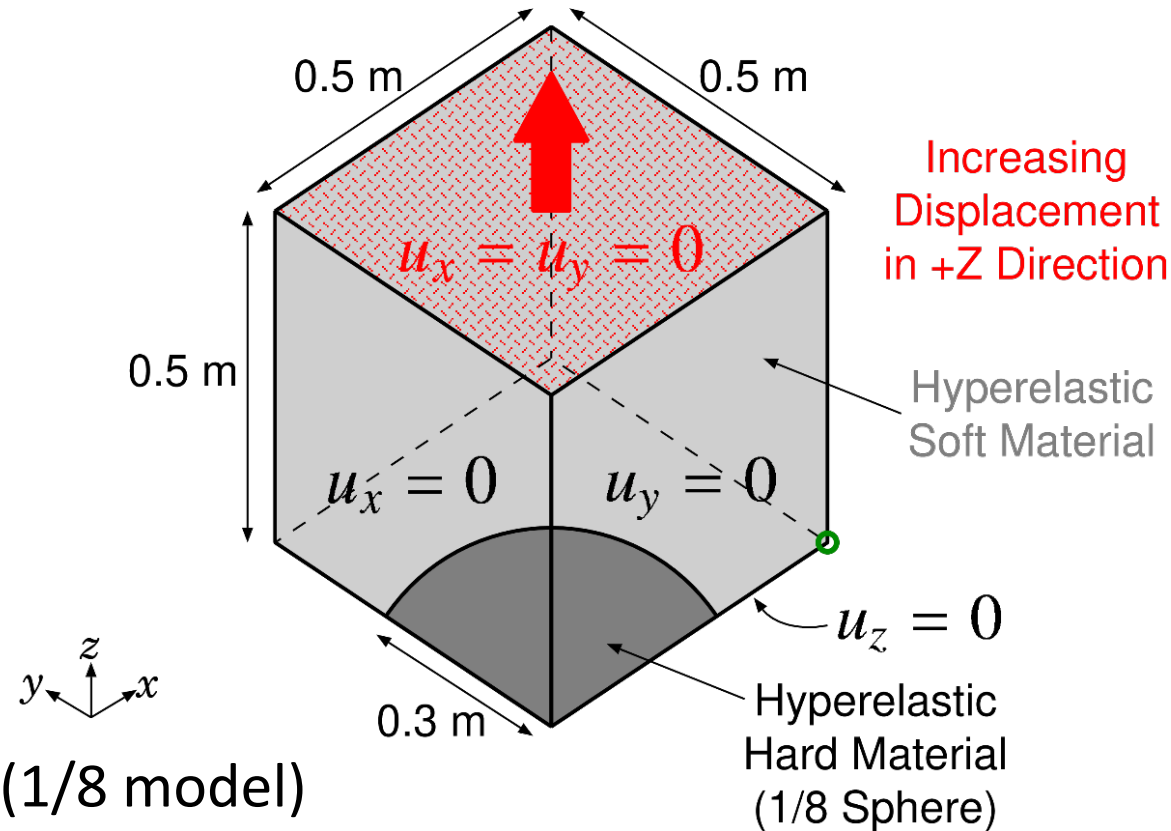
Pressure Animation of ES-FEM-SRI-T4

- No major checkerboarding.
Practical result is obtained.



Tensioning of Rubber-Filler Composite

Outline

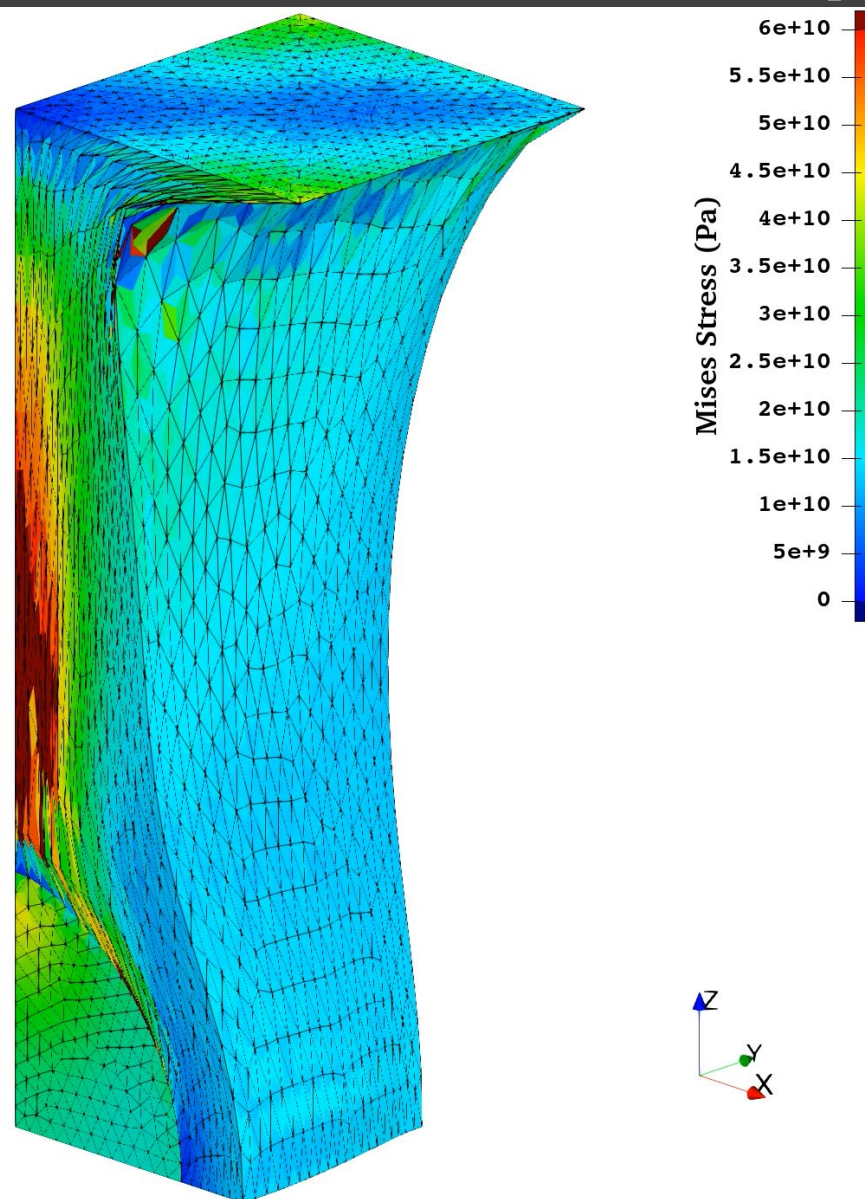


- 0.5 x 0.5 x 0.5 m cube (1/8 model)
- Rubber: Neo-Hookean hyperelastic material ($E_{ini} = 6$ GPa, $\nu_{ini} = 0.49$)
- Iron Filler: Neo-Hookean hyperelastic material ($E_{ini} = 260$ GPa, $\nu_{ini} = 0.3$)
- Applying enforced tensioning displacement on the top face with lateral confinement.
- Evaluated the result of [ES-FEM-SRI-T4](#).

Tensioning of Rubber-Filler Composite

Mises Stress Animation of ES-FEM-SRI-T4

- Smooth distribution of Mises stress.
 - Sufficient robustness in large deformation
- No major issue is found.**



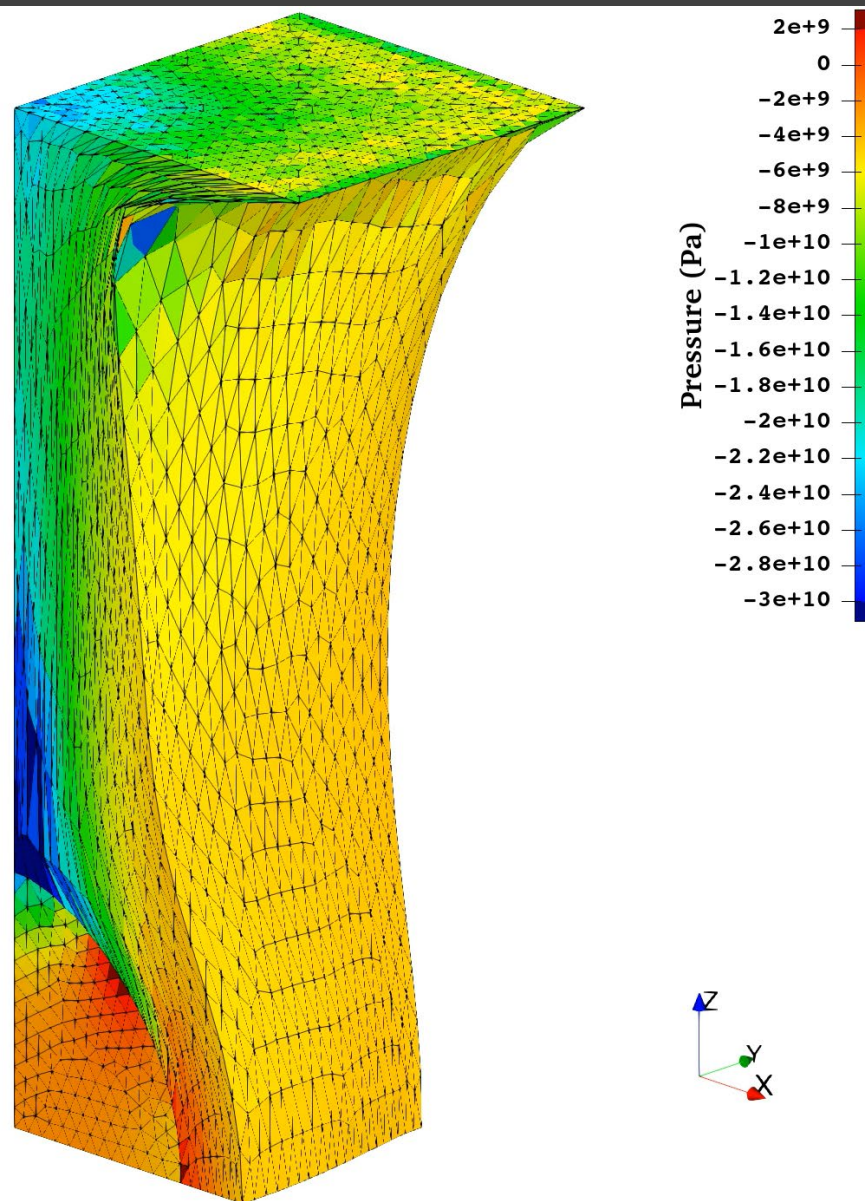
Tensioning of Rubber-Filler Composite

Pressure

Animation of

ES-FEM-SRI-T4

- No checkerboarding in the steel part.
 - No major checkerboarding in the rubber part.
- Practical result is obtained.**

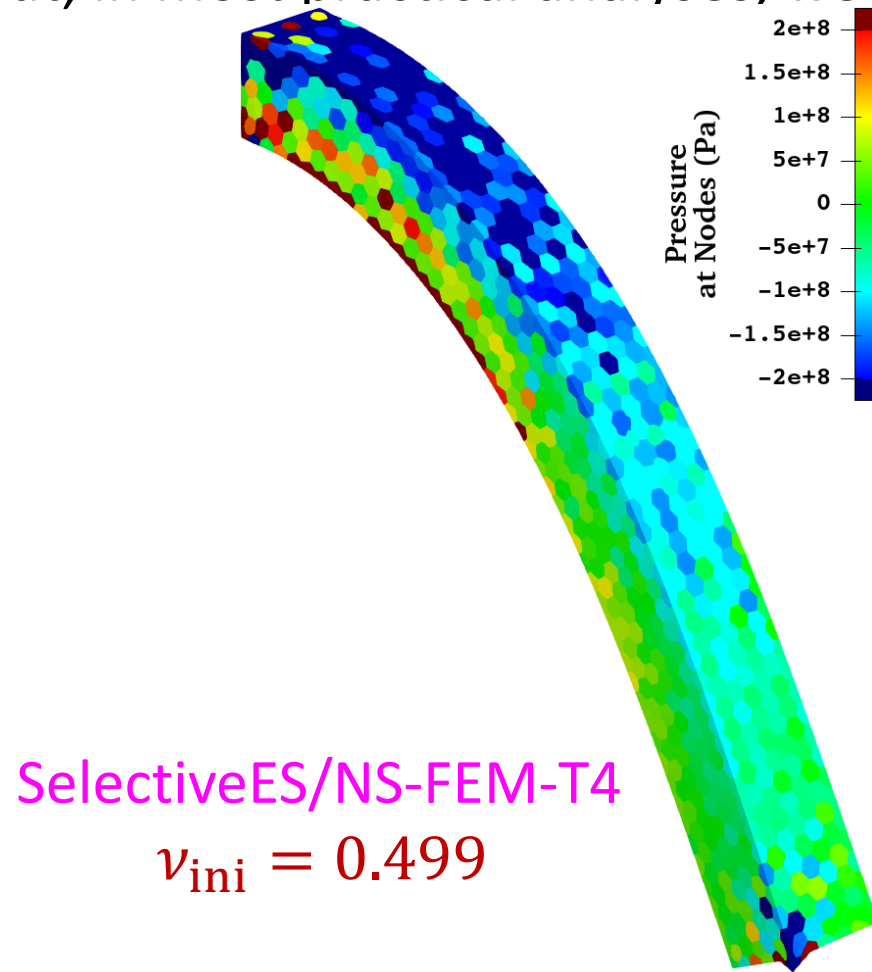


Result & Discussion 2

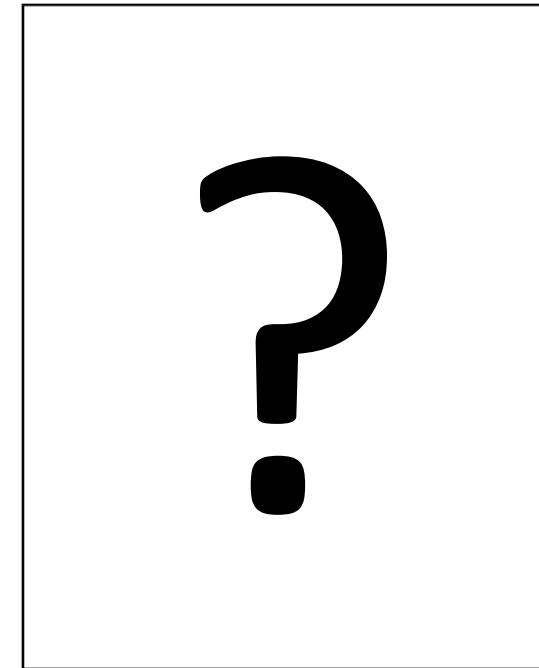
Performance of SelectiveES/NS-FEM-T4 with Post-Smoothing in Pressure

Quick Introduction to Post-Smoothing in Pressure

- The conventional S-FEM (SelectiveES/NS-FEM-T4) has the **pressure checkerboarding** issue in **nearly incompressible** analyses.
- But, in most practical analyses, we can accept results **smoothed** in post-processing.



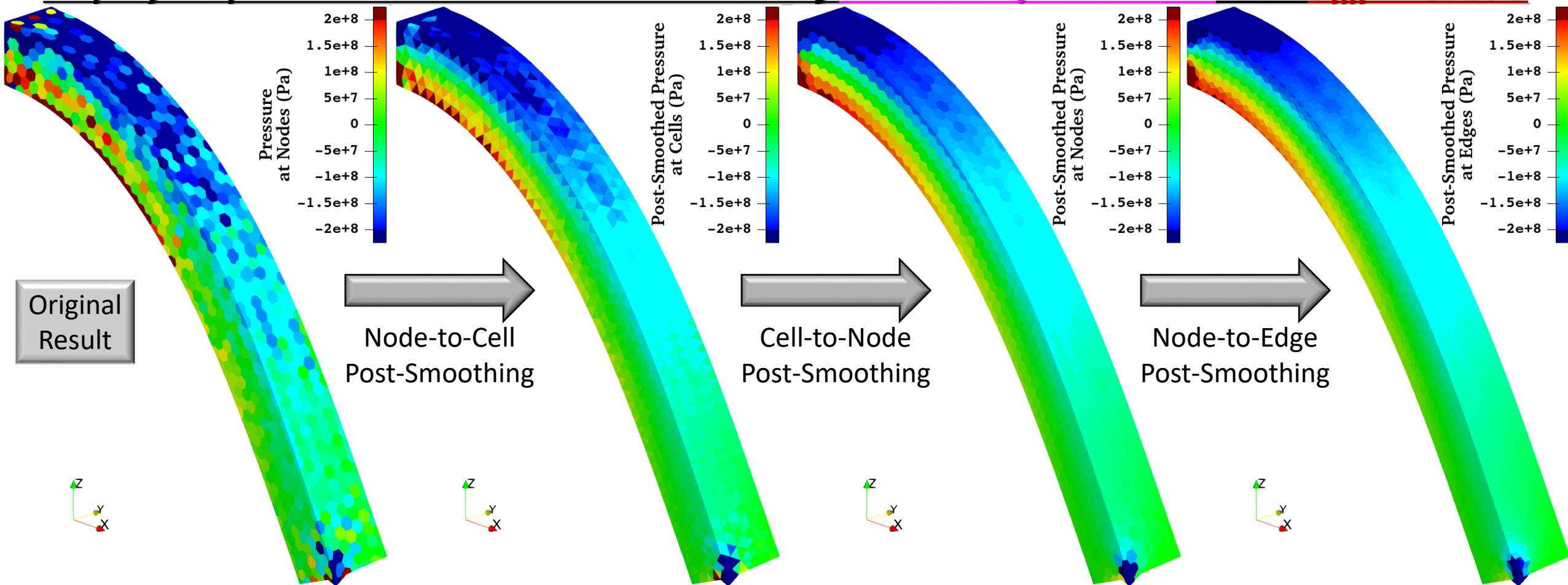
Post-Smoothing
in Pressure



Can we accept it, or not?

Bending of Rubber Cantilever

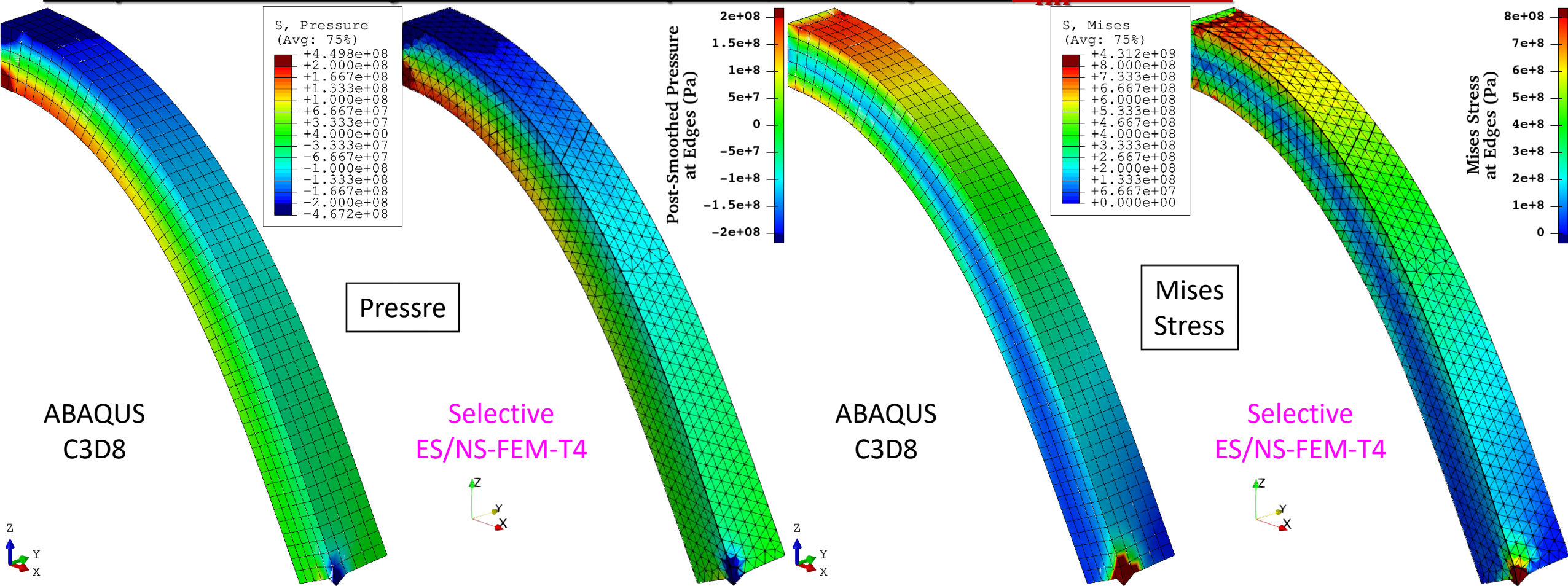
Step-by-Step Smoothed Pressure Distributions of *SelectiveES/NS-FEM-T4* with $v_{ini} = 0.499$



SelectiveES/NS-FEM-T4 with post-smoothing in pressure works so well to obtain a smooth pressure distribution.

Bending of Rubber Cantilever

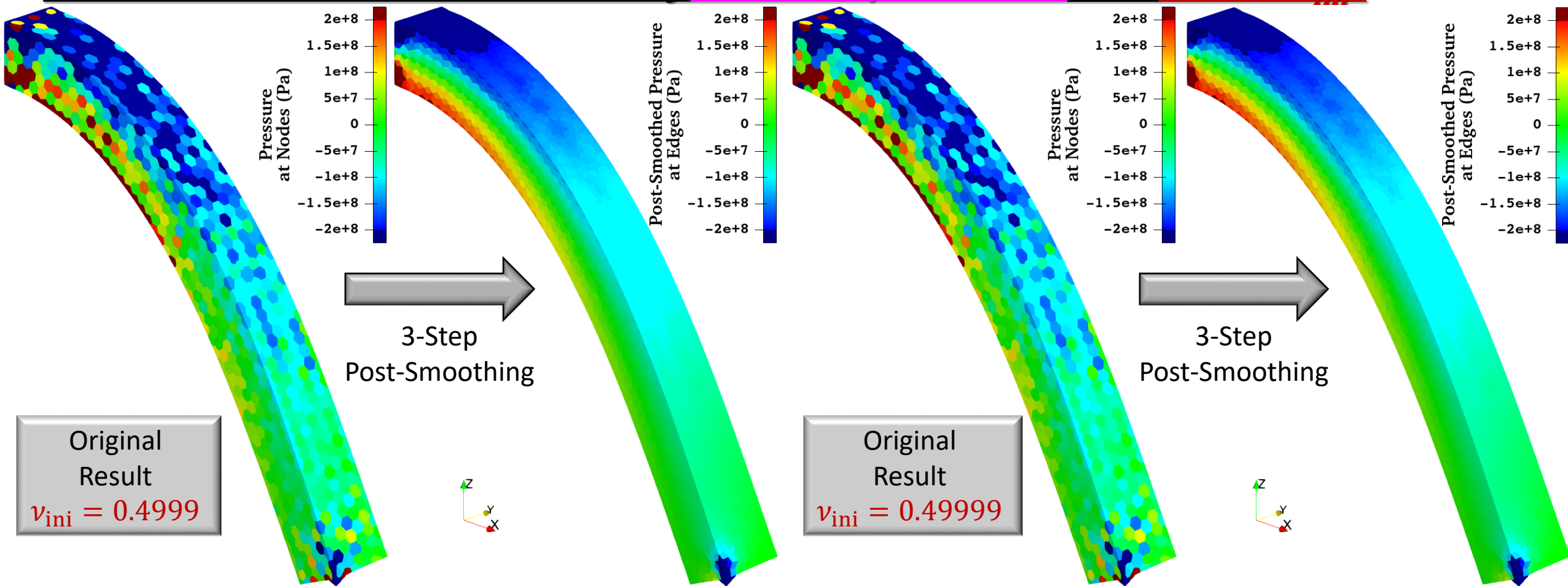
Comparison to the Reference Solution (ABAQUS C3D8) with $\nu_{ini} = 0.499$



SelectiveES/NS-FEM-T4 with post-smoothing in pressure gives acceptable solutions for all stress components.

Bending of Rubber Cantilever

Post-Smoothed Pressure Distribution of *SelectiveES/NS-FEM-T4* with *Various ν_{ini}*



SelectiveES/NS-FEM-T4 with post-smoothing in pressure works so well even for the almost incompressible cases.

Overview of the Differences Between the Two Methods

■ ES-FEM-SRI-T4:

Pre-smoothing of volumetric strain (ε_{vol}) **before** stress calculation.

→ **Pressure-dependent constitutive models** (e.g., Gurson, Drucker-Prager, Cam-Clay, etc.) **can** be treated appropriately.

■ Selective-ES/NS-FEM-T4 with Post-Smoothing in Pressure:

Post-smoothing of pressure (p, σ_{hyd}) **after** stress calculation.

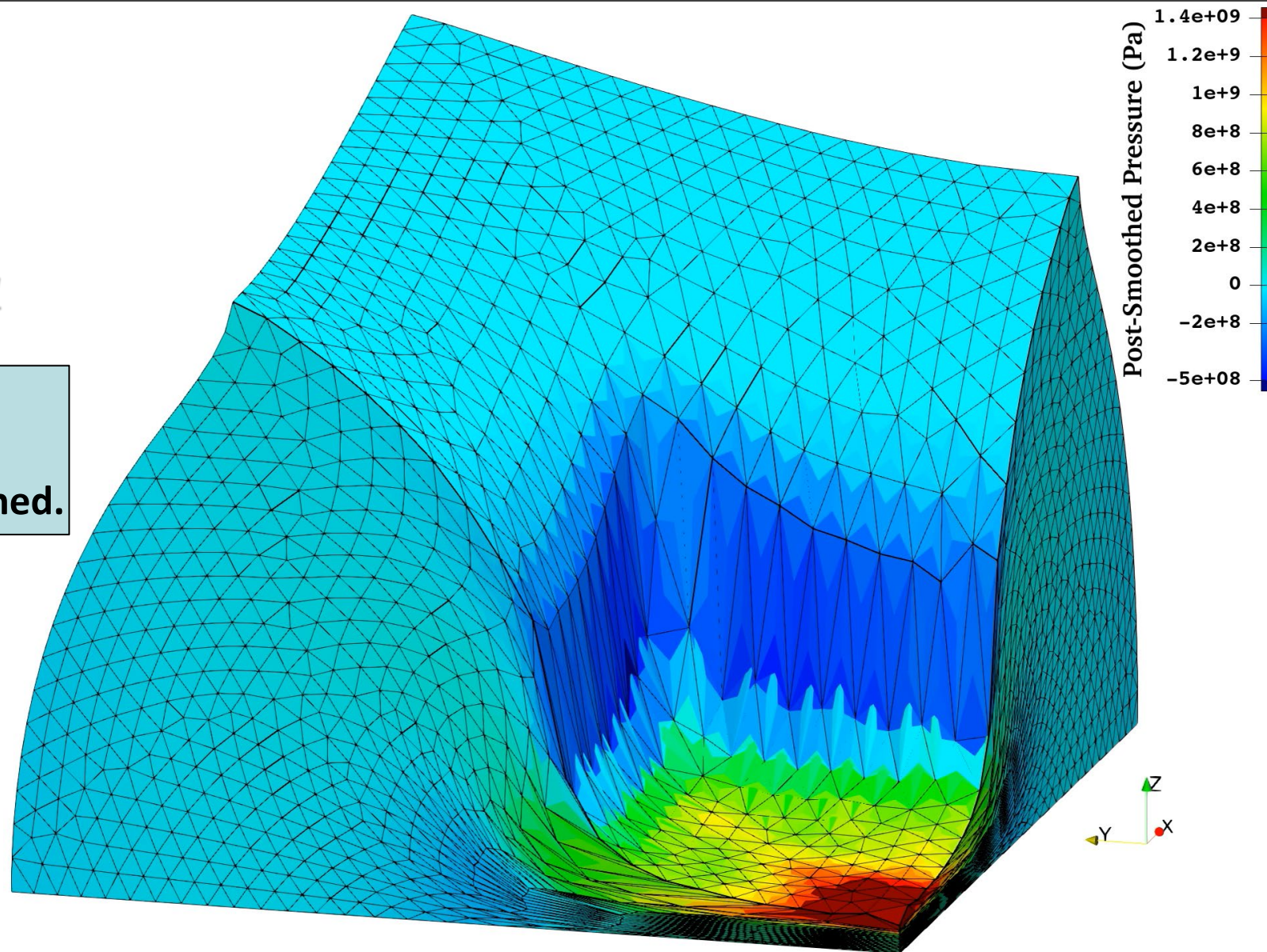
→ Pressure-dependent constitutive models **cannot** be treated appropriately.

However, since handling pressure-dependent materials is relatively rare in practice,
the post-smoothing method is sufficient in most cases.

Pressuring of Rubber Block

Pressure
Animation of
Selective
ES/NS-FEM-T4
with Post-Smoothing

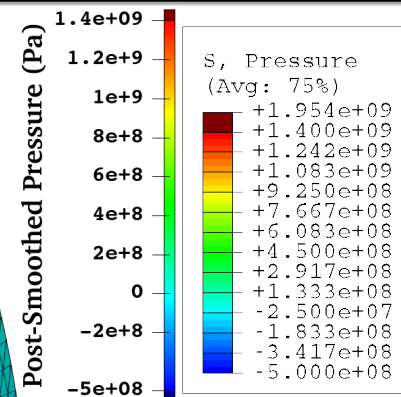
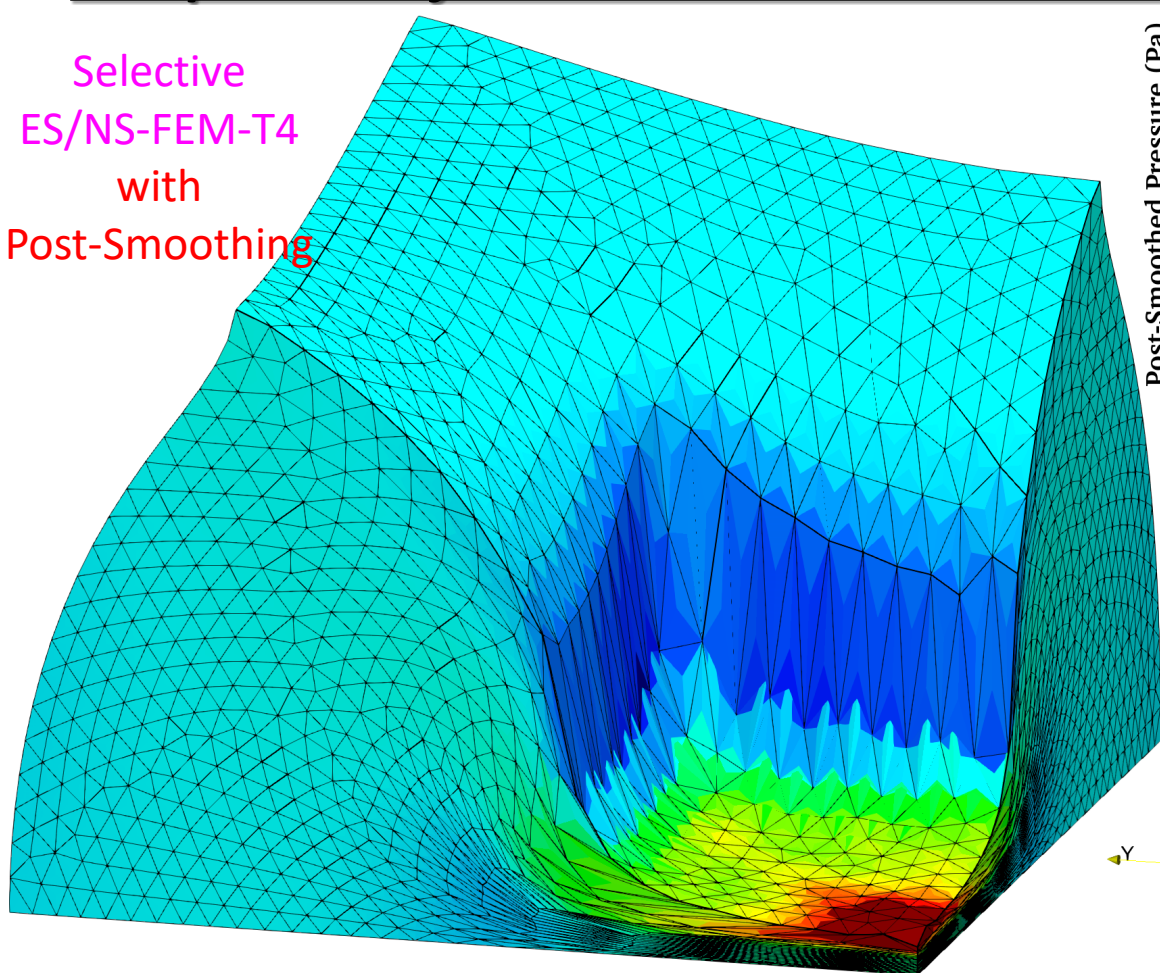
- No checkerboarding although $v_{ini} = 0.49$.
An excellent result is obtained.



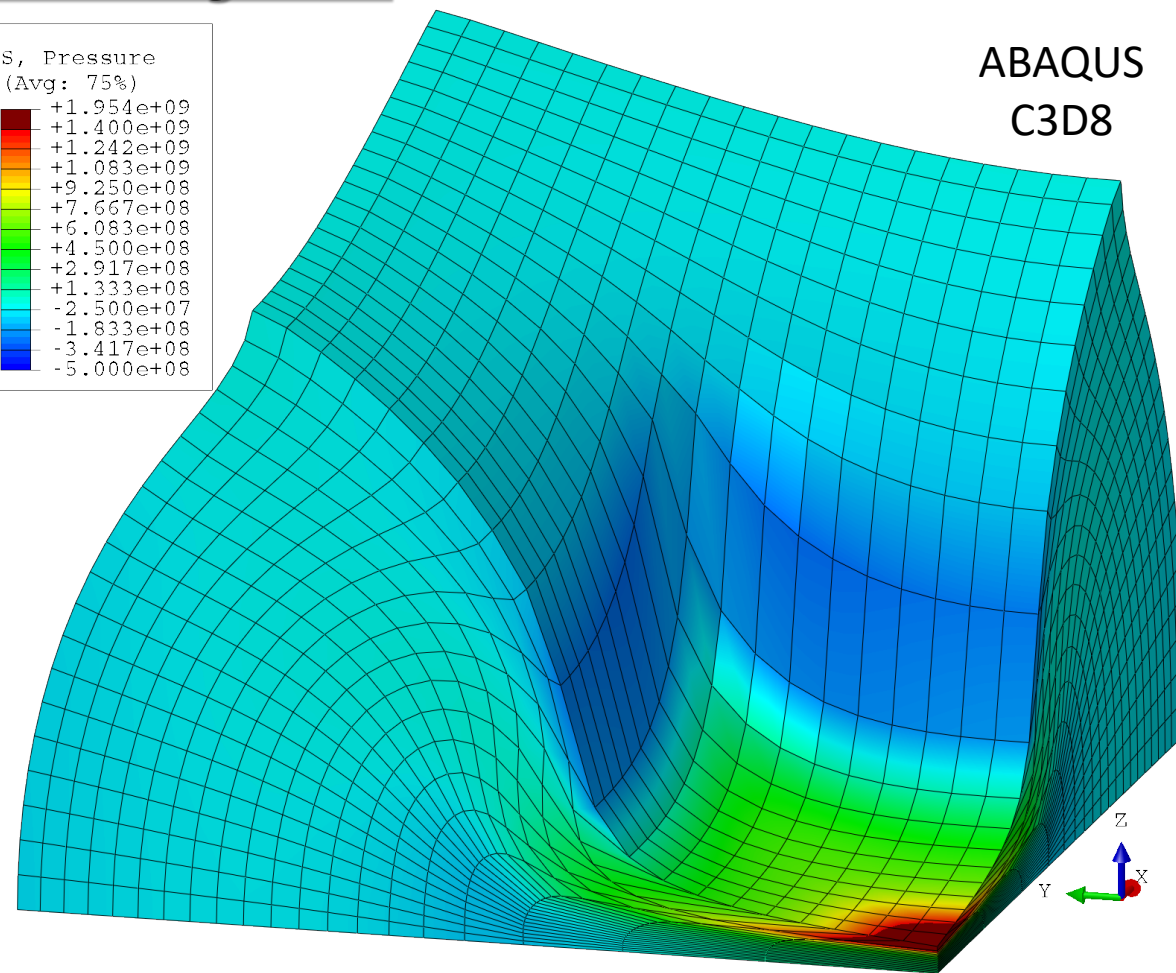
Pressuring of Rubber Block

Comparison of Pressure Distributions at 1.8 GPa Loading State

Selective
ES/NS-FEM-T4
with
Post-Smoothing



ABAQUS
C3D8



Selective ES/NS-FEM-T4 with post-smoothing in pressure works so well
in nearly incompressible **severe large deformation** analyses.

Tensioning of Rubber-Filler Composite

Pressure

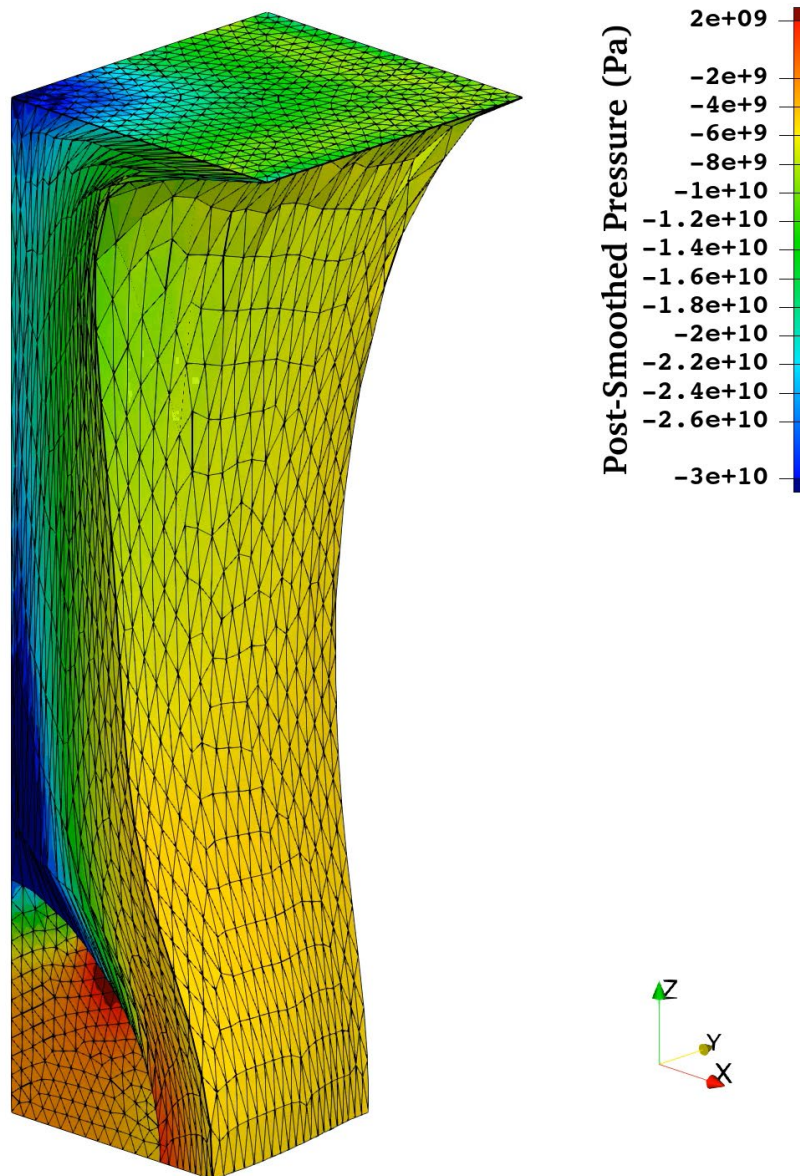
Animation of

Selective

ES/NS-FEM-T4

with Post-Smoothing

- No checkerboarding in both the rubber part ($\nu_{\text{ini}} = 0.49$) and the steel part ($\nu_{\text{ini}} = 0.3$). An excellent result is obtained.



Summary

Summary

- A new smoothed finite element method (ES-FEM-SRI-T4) was proposed.
- Despite using 4-node tetrahedral (T4) meshes, ES-FEM-SRI-T4 is not only locking-free but also pressure-checker-free for solids with Poisson's ratio up to 0.49.
- The pressure-checker-free property is especially important in the implicit analyses with pressure-dependent constitutive models (e.g., Gurson, Drucker-Prager, Cam-Clay, etc.), in which cosmetic post-smoothing in pressure does not work.
- For the analyses only with pressure-independent constitutive models, the conventional SelectiveES/NS-FEM-T4 with post-smoothing in pressure works well.

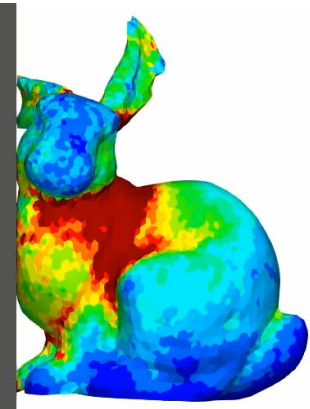
Take Home Message:

By using SelectiveES/NS-FEM-T4 with post-smoothing or ES-FEM-SRI-T4 appropriately, most practical problems, including nearly incompressible large deformation analyses with complex geometries, are accurately solved.

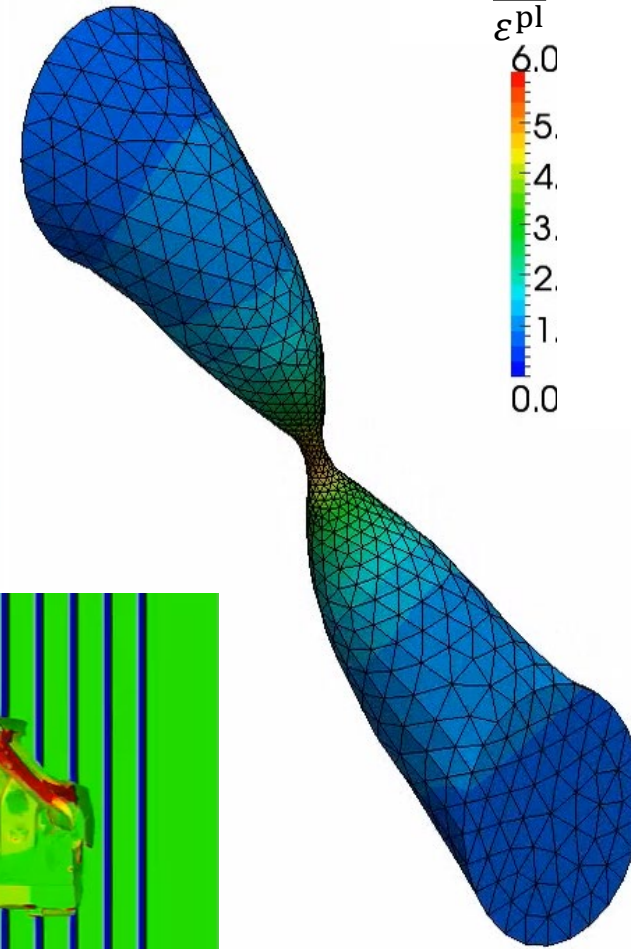
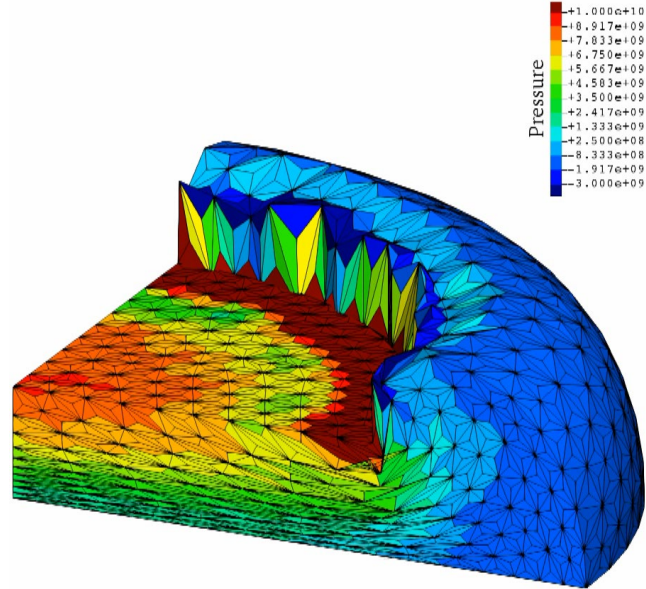
Applications of S-FEM-T4s in Our Lab

■ Large deformation solid mechanics (still in academic research)

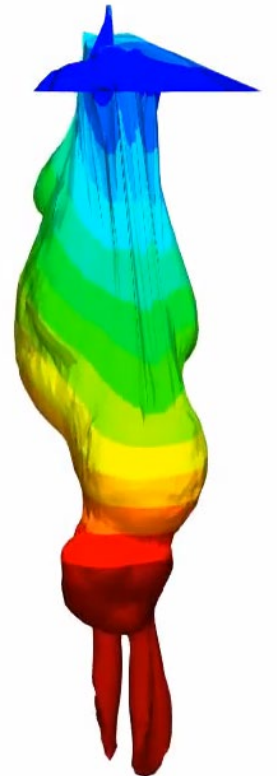
Dynamic Explicit



Static Implicit

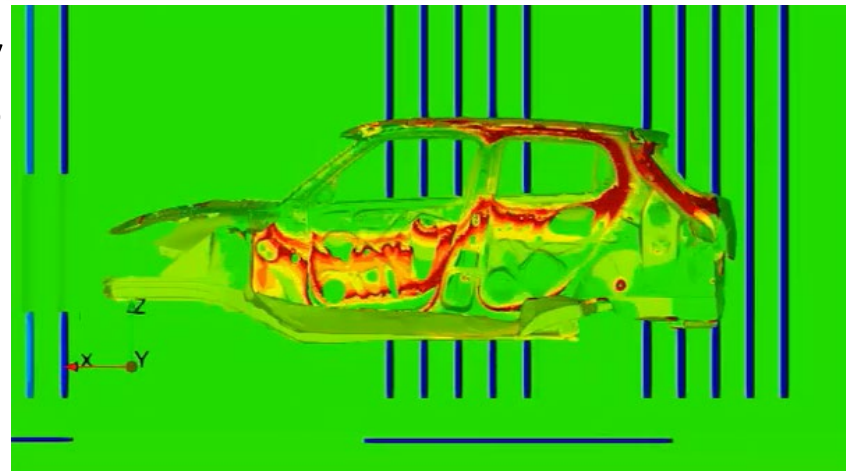


Viscous Implicit



■ Electrostatic (already in commercial use)

*Carbody
Electro
Deposition*



Thank you for your
kind attention!

Appendix

Proper Usage of S-FEM-T4s

Material Properties	Recommended Method
A) Compressible	● ES-FEM-T4 or ● SelectiveES/NS-FEM-T4
B) Incompressible & Pressure-independent	● SelectiveES/NS-FEM-T4 with Post-Smoothing
C) Incompressible & Pressure-dependent	● ES-FEM-SRI-T4 with Poisson's Ratio up to 0.49 or ● SelectiveES/NS-FEM-T4 with Post-Smoothing and Explicit parameterization for Pressure

Post-smoothing is a technique for cosmetic improvement and is not welcomed; however, it is an extremely useful technique for materials without pressure dependency in engineering applications.